Recupero 2011-12

1. Numeri complessi

1.1 Espressione da semplificare

es. $\frac{3-5i}{2+i} \cdot \frac{4-2i}{1+i}$

1.2 Equazione a coefficienti complessi

es. $z^2 - iz + 2 = 0$

2. Topologia

2.1 Es. $A = \left\{ x \mid x = \frac{1-m}{2m},\ m \in \mathbb{N}^* \right\}$

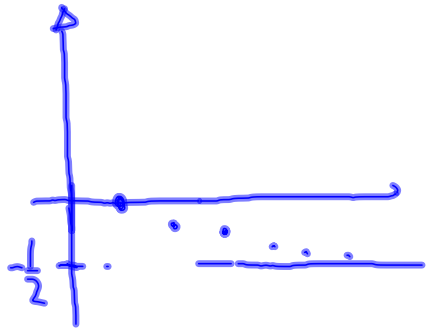
Stabilire:

1) Limito / illim.
2) Estremo Sup / inf
   max. min.
3) P.ti accumulazione
4) Dim. pto 1.

$x = \frac{1-m}{2m} = \frac{1}{2m} - \frac{1}{2}$

$\frac{1}{2}\left(\frac{1}{m} - 1\right)$

$A = \left\{ 0, -\frac{1}{4}, -\frac{1}{3}, \dots, \dots \right\} \to -\frac{1}{2}$



0 è un maggiorante

cioè $0 \geq x \quad \forall x \in A$

dim. $x \in A$ e $x \leq 0$

$-\frac{1}{2}$ è un minorante

$-\frac{1}{2} < \frac{1}{2}\left(\frac{1}{m} - 1\right) \quad \forall m$

$-x' < \frac{1}{n} - x$

$0 < \frac{1}{m} \quad \forall m > 0$

3. Successioni

$$f: \mathbb{N} \to \mathbb{R}$$

$$\{a_n\}_{n \in \mathbb{N}}$$

es. $\left\{\frac{1}{n-1}\right\}_{n \in \mathbb{N}-\{0,1\}} = \left\{1, \frac{1}{2}, \frac{1}{3}, \dots\right\}$

$$\lim_{n \to +\infty} \frac{1}{n-1} = 0$$

infatti:

$\forall \varepsilon > 0$ per $\exists \bar{n} \mid \forall n > \bar{n}$

$$\left| \frac{1}{n-1} - 0 \right| < \varepsilon \qquad n \geq 2$$

$$\frac{1}{n-1} < \varepsilon \qquad n - 1 > \frac{1}{\varepsilon}$$

$$n > 1 + \frac{1}{\varepsilon}$$

$$\bar{n} = \left\lceil 1 + \frac{1}{\varepsilon} \right\rceil \; //$$

4. Limiti (definizione)

4.1 $\lim_{x\to 1} \frac{x}{x+2} = \frac{1}{3}$

$\forall \varepsilon > 0 \ \exists U_1 \mid \forall x \in U_1 \cap D - \{1\}$

$\left|\frac{x}{x+2} - \frac{1}{3}\right| < \varepsilon$

4.2 $\lim_{x\to -2} \frac{x}{x+2} = \infty$

$\forall M > 0$ qqp $\exists U_{-2} \mid \forall x \in U_{-2} \cap D - \{-2\}$

$\left|\frac{x}{x+2}\right| > M$

4.3 $\lim_{x\to 3} \frac{1}{(x-3)^2} = +\infty$

$\forall M > 0$ qqp $\exists U_3 \mid \forall x \in U_3 \cap D - \{3\}$

$\frac{1}{(x-3)^2} > M$

4.4 $\lim_{x\to +\infty} 3^{-x} = 0$

$\forall \varepsilon > 0 \ \exists U_{+\infty} \mid \forall x \in U_{+\infty} \cap D$

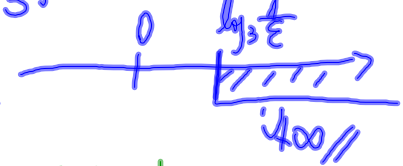
$|3^{-x} - 0| < \varepsilon$

$3^{-x} < \varepsilon$

$\frac{1}{3^x} < \varepsilon$

$3^x > \frac{1}{\varepsilon} = 3^{\log_3 \frac{1}{\varepsilon}}$

$x > \log_3 \frac{1}{\varepsilon}$



4.5 $\lim_{x\to -\infty} \ln|x+1| = +\infty$

$\forall M > 0 \ \exists U_{-\infty} \mid \forall x \in U_{-\infty} \cap D$

$\ln|x+1| > M$

$\ln|x+1| > \ln e^M$

$|x+1| > e^M$

$x+1 > e^M \quad x > e^M - 1$

$\cup$

$x+1 < -e^M \quad x < -e^M - 1$