Recupero 2011-12

1. Numeri complessi

1.1. Espressione da semplificare

es. $\frac{3-5i}{2+i} \cdot \frac{4-2i}{1+i}$

1.2 Equazione a coefficienti complessi

es. $z^2 - iz + z = 0$

2. Topologia

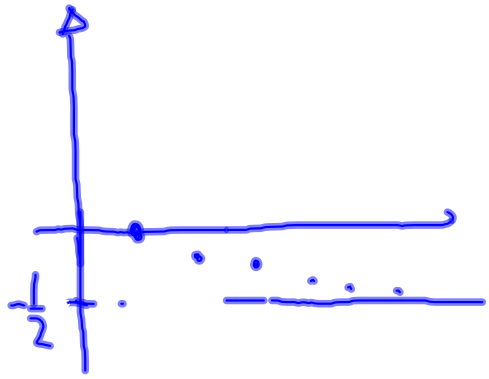
2.1 Es. $A = \{ x \mid x = \frac{1-n}{2n}, n \in \mathbb{N}^* \}$

Stabilire:

1) Limito / illim.
2) Estremo Sup / inf
   max. min.
3) P.ti accumulazione
4) Dim. pto 1.

$x = \frac{1-n}{2n} = \frac{1}{2n} - \frac{1}{2} = \frac{1}{2}\left(\frac{1}{n} - 1\right)$

$A = \{ 0, -\frac{1}{4}, -\frac{1}{3}, \dots \} \to -\frac{1}{2}$



0 è un maggiorante

cioè $0 \geq x \quad \forall x \in A$

dim. $x \in A$ e $x \leq 0$

$-\frac{1}{2}$ è un minorante

$$-\frac{1}{2} < \frac{1}{2}\left(\frac{1}{n} - 1\right) \quad \forall n$$

$$-\cancel{1} < \frac{1}{n} - \cancel{1}$$

$$0 < \frac{1}{n} \quad \forall n > 0$$

3. Successioni

$\{a_n\}_{n \in \mathbb{N}}$ $\qquad f: \mathbb{N} \to \mathbb{R}$

es. $\left\{\frac{1}{n-1}\right\}_{n \in \mathbb{N} - \{0,1\}} = \left\{1, \frac{1}{2}, \frac{1}{3}, \dots\right\}$

$$\lim_{n \to +\infty} \frac{1}{n-1} = 0$$

infatti:

$\forall \varepsilon > 0$ pop $\exists \bar{n} \mid \forall n > \bar{n}$

$$\left| \frac{1}{n-1} - 0 \right| < \varepsilon \qquad n \geq 2$$

$\frac{1}{n-1} < \varepsilon \qquad n-1 > \frac{1}{\varepsilon}$

$n > 1 + \frac{1}{\varepsilon}$

$\bar{n} = \left\lceil 1 + \frac{1}{\varepsilon} \right\rceil$ //

4. Limiti (definizione)

4.1 $\lim\limits_{x\to 1} \frac{x}{x+2} = \frac{1}{3}$

$\forall \varepsilon>0\ \exists U_1 \mid \forall x\in U_1 \cap D-\{1\}$

$\left|\frac{x}{x+2}-\frac{1}{3}\right|<\varepsilon$

4.2 $\lim\limits_{x\to -2} \frac{x}{x+2} = \infty$

$\forall M>0$ gap $\exists U_{-2} \mid \forall x\in U_{-2}\cap D-\{-2\}$

$\left|\frac{x}{x+2}\right|>M$

4.3 $\lim\limits_{x\to 3} \frac{1}{(x-3)^2} = +\infty$

$\forall M>0$ gap $\exists U_3 \mid \forall x\in U_3\cap D-\{3\}$

$\frac{1}{(x-3)^2}>M$

4.4 $\lim\limits_{x\to +\infty} 3^{-x} = 0$

$\forall \varepsilon>0\ \exists U_{+\infty} \mid \forall x\in U_{+\infty}\cap D$

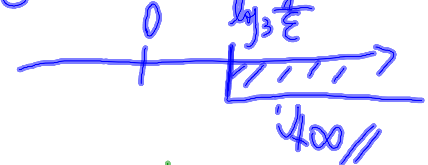
$|3^{-x}-0|<\varepsilon$

$3^{-x}<\varepsilon$

$\frac{1}{3^x}<\varepsilon$

$3^x > \frac{1}{\varepsilon} = 3^{\log_3 \frac{1}{\varepsilon}}$

$x > \log_3 \frac{1}{\varepsilon}$



4.5 $\lim\limits_{x\to -\infty} \ln|x+1| = +\infty$

$\forall M>0\ \exists U_{-\infty} \mid \forall x\in U_{-\infty}\cap D$

$\ln|x+1|>M$

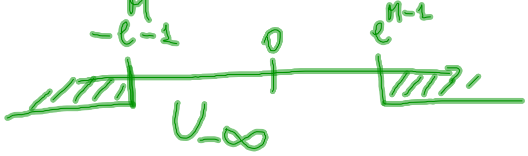
$\ln|x+1|>\ln e^M$

$|x+1|>e^M$

$x+1>e^M \quad x>e^M-1$

$\cup$

$x+1<-e^M \quad x<-e^M-1$



$$\frac{5+i}{1+2i}+\frac{1+i}{1-2i}$$

$$\frac{(5+i)(1-2i)+(1+i)\cdot(1+2i)}{(1+2i)(1-2i)}$$

$$\frac{5-10i+i-\cancel{2i^2}+1+2i+i+\cancel{2i^2}}{(1+2i)(1-2i)}$$

$$\frac{-6i+6}{(1+2i)(1-2i)}=\frac{-6(i-1)}{(1+2i)(1-2i)}=\frac{-6(i-1)}{1-4i^2}$$

$$\frac{-6(i-1)}{5}$$

$a+ib$

$\frac{6}{5}-\frac{6}{5}i$

$$\frac{1+i}{1-2i}\cdot\frac{1+2i}{1+2i}$$

$$\frac{1+2i+i+2i^2}{5}=\frac{-1+3i}{5}$$

$-\frac{1}{5}+\frac{3}{5}i$

$$|z|=\sqrt{a^2+b^2}$$
$$=\sqrt{1/25+\frac{9}{25}}$$
$$=\sqrt{10/25}=\sqrt{\frac{2}{5}}$$

$1+i$ $\sqrt{1+1}=\sqrt{2}$

$a+ib$

$\arctan\frac{b}{a}=1$

$$\sqrt{2}\cdot\left(\cos\frac{\pi}{4}+i\,\mathrm{sen}\frac{\pi}{4}\right)$$

$$\sqrt{2}\cdot e^{i\frac{\pi}{4}}$$

$$e^{i\frac{\pi}{2}}$$

$$\left(\cos\frac{\pi}{2}+i\,\mathrm{sen}\frac{\pi}{2}\right)$$

$$(0+i1)=i$$

$$z^2 - iz + 2 = 0$$

$$\Delta = -1 - 8 = -9$$

$$z_{1,2} = \frac{i \pm \sqrt{-9}}{2} = \frac{i-3i}{2} = \frac{-2i}{2} = -i$$

$$\frac{i+3i}{2} = \frac{4i}{2} = 2i$$

$$\sqrt{9} = \sqrt{-1 \cdot 9} = 3\sqrt{-1} = \begin{cases} +3i \\ -3i \end{cases}$$

$$\frac{i+3i}{2} = \frac{4i}{2} = 2i$$

$$\frac{i-3i}{2} = \frac{-2i}{2} = -i$$

$$\sqrt[3]{i}$$

$$|z| = 1$$

$$\left(\cos\frac{\pi}{2} + i\,\mathrm{sen}\,\frac{\pi}{2}\right)$$

$$\sqrt[3]{e^{i\frac{\pi}{2}}} = e^{i\frac{\pi}{6} + 2k\frac{\pi}{3}}$$

$$k \to 0 \Rightarrow e^{i\frac{\pi}{6}}$$

$$k \to 1 \Rightarrow e^{i\frac{\pi}{6} + \frac{2\pi}{3}} = e^{i\frac{5\pi}{6}}$$

$$k \to 2 \Rightarrow e^{i\frac{\pi}{6} + \frac{4\pi}{3}} = e^{i\frac{9\pi}{6}} = e^{i\frac{3\pi}{2}}$$

$$A = \left\{ \frac{n}{n^2+10} \right\} \qquad \frac{1}{n + \frac{10}{n}}$$

$$= \left\{ 0, \frac{1}{11}, \frac{2}{\cancel{14}}\frac{1}{7}, \frac{3}{19}, \dots \frac{11}{131} \dots \right\}$$

0 MINIMO EST. INF.

MASSIMO EST. SUP.

1

$A_{cc}(A) = 0$

$0 \leq \frac{n}{n^2+10} \quad \forall n$

sempre positivo

$n \geq 0$

dim.

$$A = [0, 1] \cup \{3\}$$

minimo massimo

est. inf. est. sup

$$Acc(A) = [0, 1]$$

$$]3-\varepsilon, 3+\varepsilon[$$

$$\varepsilon < 2$$

$$\lim_{n\to+\infty} 3+\sqrt{n} = +\infty$$

$\forall M>0$ gof, $\exists \bar{n} \mid \forall n>\bar{n} \Rightarrow a_n > M$

$3+\sqrt{n} > M$

$\sqrt{n} > M-3$

$n > M^2-6M+9 \quad \bar{n} = \lceil M^2-6M+9 \rceil$

$$\lim_{n\to+\infty} (-1)^n \frac{3n}{n+1} = \text{indeterminato}$$

$$\lim_{x\to+\infty} \frac{3x}{x+1} = \lim_{x\to+\infty} \frac{\frac{3x}{x}}{\frac{x}{x}+\frac{1}{x}} = \frac{3}{1+\frac{1}{x}}$$

$$\lim_{x \to +\infty} \frac{3x}{x+1} = 3$$

$\forall \varepsilon > 0$ pay $\exists U_{+\infty} \mid \forall x \in U_{+\infty} \cap D \Rightarrow$

$$\left| \frac{3x}{x+1} - 3 \right| < \varepsilon$$

$$\begin{cases} \frac{3x-3x-3}{x+1} = \frac{-3}{x+1} < \varepsilon \\ \frac{-3}{x+1} > -\varepsilon \end{cases}$$

$$\frac{-3}{x+1} < \varepsilon \Rightarrow \forall x \quad x > 0$$

$$\frac{-3}{x+1} > -\varepsilon \quad \frac{3}{x+1} < \varepsilon$$

$$3 < x\varepsilon + \varepsilon$$

$$x\varepsilon > 3 - \varepsilon$$

$$x > \frac{3-\varepsilon}{\varepsilon} \cap x > 0 \Rightarrow \boxed{x > \frac{3-\varepsilon}{\varepsilon}} = U_{+\infty}$$

$$\lim_{x \to -\infty} \frac{1}{3^x - 1} = -1$$

$\forall \varepsilon > 0$ per $\exists U_{-\infty} \mid \forall x \in U_{-\infty} \cap D \Rightarrow$

$$\left| \frac{1}{3^x - 1} + 1 \right| < \varepsilon$$

$$\begin{cases} \dfrac{1 + 3^x - 1}{3^x - 1} = \dfrac{3^x}{3^x - 1} < \varepsilon \\ \dfrac{3^x}{3^x - 1} > -\varepsilon \end{cases}$$

$$\frac{3^x}{3^x - 1} < \varepsilon \Rightarrow \forall x \quad x < 0$$

$$\frac{3^x}{3^x - 1} > -\varepsilon$$

$$\frac{3^x}{3^x - 1} + \varepsilon > 0$$

$$\frac{3^x + \varepsilon 3^x - \varepsilon}{3^x - 1} > 0 \Rightarrow 3^x + \varepsilon 3^x - \varepsilon < 0$$

$$3^x (1 + \varepsilon) - \varepsilon < 0$$

$$3^x < \frac{+\varepsilon}{1 + \varepsilon}$$

$$\log_3 3^x < \log_3 \frac{\varepsilon}{1 + \varepsilon}$$

$$x < \log_3 \frac{\varepsilon}{1 + \varepsilon} = U_{-\infty}$$

$$\lim_{x \to +\infty} \left(\sqrt{x(x+2)} - x\right)$$

$$\frac{\left(\sqrt{x(x+2)} - x\right) \cdot \left(\sqrt{x(x+2)} + x\right)}{\sqrt{x(x+2)} + x}$$

$$\frac{x(x+2) - x^2}{\sqrt{x(x+2)} + x} = \frac{2x}{\sqrt{x(x+2)} + x}$$

$$\frac{\frac{2x}{x}}{\frac{\sqrt{x(x+2)} + x}{x}}$$

$$\frac{2}{\frac{\sqrt{x(x+2)}}{x} + 1}$$

$$\frac{2}{\frac{\sqrt{x(x+2)}}{\sqrt{x^2}} + 1}$$

$$\frac{2}{\sqrt{\frac{x(x+2)}{x^2}} + 1} = \frac{2}{\sqrt{\frac{x+2}{x}} + 1}$$

$$\frac{2}{\sqrt{1+\frac{2}{x}} + 1} = 1$$

$$\lim_{x \to +\infty} \left(\sqrt{x(x+2)} - x \right) = 1$$

$\forall \varepsilon > 0$ per $\exists U_{+\infty} \mid \forall x \in U_{+\infty} \cap D \Rightarrow$

$$\left| \sqrt{x(x+2)} - x - 1 \right| < \varepsilon$$

$$\begin{cases} \sqrt{x(x+2)} - x - 1 < \varepsilon \quad \dots\dots \\ \end{cases}$$

$$\lim_{x \to 2} \frac{x+1}{x-2} = \infty$$

$\forall M > 0$ gas $\exists U_2 \mid \forall x \in U_2 \cap D - \{2\}$

$$\left| \frac{x+1}{x-2} \right| > M$$

$$\frac{x+1}{x-2} > M$$

$$\frac{x+1-Mx+2M}{x-2} > 0$$

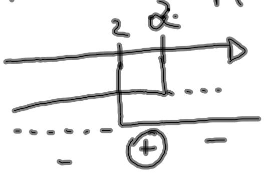
$$x+1-Mx+2M > 0$$

$$x - Mx > -1-2M$$

$$x(1-M) > -1-2M$$

$$x < \frac{-1-2M}{1-M} \Rightarrow x < \frac{1+2M}{M-1} = \frac{\frac{1}{M}+2}{1-\frac{1}{M}} > 2$$

$$x > 2$$



$$2 < x < \alpha$$

$$\frac{x+1}{x-2} < -M$$

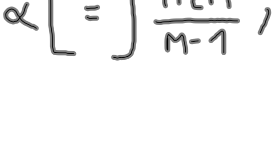
$$\frac{x+1+Mx-2M}{x-2} < 0$$

$$x+1+Mx-2M > 0$$

$$x(1+M) > 2M-1$$

$$x > \frac{2M-1}{1+M} = \frac{2-\frac{1}{M}}{\frac{1}{M}+1} < 2$$

$$x > 2$$



$$U_2 = \left]\beta, \alpha\right[= \left] \frac{1+2M}{M-1}, \frac{2M-1}{1+M} \right[$$